DESIGN EQUATIONS FOR DYNAMITRON TYPE POWER SUPPLIES IN THE MEGAVOLT RANGE

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SUMMARY

The power supply which is used as the accelerating potential for megavolt Dynamitron accelerators is a parallel-fed, series-cascaded rectifier system. Low frequency RF voltage is capacitively coupled into the cascaded rectifier system by means of an SF-6 gas insulated electrode configuration. Maximum voltage attained with this configuration has been 5.7 MV. Maximum current attained has been 20 mA. Maximum beam power attained has been 40 kW. This paper covers the development of equations which express the operational input parameters such as, oscillator plate voltage, current, and power in terms of the required output high voltage and load current, and the internal circuit parameters such as, tank capacitance, tank inductance resistance, number of rectifier stages, and oscillating frequency. Calculated data from these equations agree closely with experimental data. This analysis allows the optimization of a power supply design in terms of critical parameters for a given maximum output voltage and load current.

INTRODUCTION

The power supply is used as the accelerating potential for megavolt Dynamitron ion and electron accelerators. The unique features of the power supply are: 1) the capability to supply high beam power, 2) the low stored energy of the system, 3) the inherent voltage stability, and 4) the simplicity of the assembly. Accelerators have been or are being designed and manufactured covering the range of voltages from 1.0 MV to 5.5 MV, and beam currents of 25 mA to 3.5 mA respectively. The maximum current attained has been 20 mA and the maximum beam power 40 kW.

Equations have been developed which express the operational input parameters such as, oscillator plate voltage, current and power in terms of the required output voltage and load current, and the internal circuit parameters such as, tank capacitance, tank transformer resistance, number of rectifier stages and the resonance frequency. These equations have been used to design a new 1.5 MV, 20 mA electron accelerator. This accelerator has been manufactured and has undergone high voltage no-load tests. At the present time the accelerator is being reassembled in a test vault for beam tests.

DESCRIPTION OF POWER SUPPLY

The high voltage generator[1] is a cascaded rectifier system, Fig. 1, whose rectifiers are parallel-fed through the inherent coupling capacitance C_{se}, created by the corona shields and RF electrodes. Low frequency RF voltage applied to the RF electrodes is generated by a high-power oscillator operating at a frequency of 112 kHz. The tank circuit is made up of a gas insulated toroidal transformer, and the total capacitance of the electrode system. The oscillator is a tuned-plate, untuned grid circuit. The oscillator tubes receive their grid drive from a plate located between the RF electrode and the pressure vessel. The high voltage is controlled by varying the amplitude of RF voltage which in turn is varied by adjusting the anode dc voltage by means of the series-pass tubes.

DEVELOPMENT OF DESIGN EQUATIONS

In the following development the reader is referred to Table I for definition of terms. The dc voltage generated per stage is a function of the capacitive elements and the peak applied RF voltage, E_f. A section of the rectifier cascade is shown in Fig. 2. By applying Kirchoff's law of potential drops around closed paths the ac voltage per stage can be shown to be

$$E_{ac} = E_f/(1 + 4\ C_{ac}/C_{se}) \qquad (1)$$

The term $1 + 4\ C_{ac}/C_{se}$ is designated as the coupling coefficient K, i.e.,

$$E_{ac} = E_f/K \qquad (2)$$

Since the dc voltage per stage is equal to the peak ac voltage per stage at a no-load condition,

$$E_o = E_f/K \qquad (3)$$

The voltage droop per stage is a function of the stored charge per stage and the charge removed per cycle of operation. If an equivalent circuit of Equation (3) is made as shown in Fig. 3, it can be observed that the stored charge is the voltage times the sum of the capacitances in the circuit, i.e.,

$$Q_s = (C_{se} + 4\ C_{ac})\ E_o = KC_{se}\ E_o$$

If the charge removed per cycle is,

$$\Delta Q_s = i\Delta t = IT = I/f.$$

also $\Delta Q_s = (C_{se} + 4\ C_{ac})\ \Delta E_o$

then $\Delta E_o = I/fKC_{se}$.

The dc voltage per stage including the droop term is $E'_o = E_o - \Delta E_o$

$$E'_o = E_f/K - I/fKC_{se}$$

The total generated dc voltage of the cascade circuit is just

$$E = NE'_o = \cdot E_f \ (N/K) - NI/fKC_{se} \quad (4)$$

because the dc and ac voltages of all stages are the same. Solving equation (4) for E_f we have,

$$E_f = E\ (K/N) + I/fC_{se} \quad (5)$$

The RF power required is

$$P_{RF} = I^2_{RF}\ Re/_2 + EI \quad (6)$$

Where the first term is the rms power loss in the RF transformer and the second term is the average load power. The RF current in the tank circuit can be expressed as

$$I_{RF} = \omega C_T E_f$$

Rewriting Equation (6) we have,

$$P_{RF} = \omega^2 C_T{}^2 R_e\ E^2{}_f/2 + EI$$

$$= \omega^2 C^2{}_T\ R_e A^2\ (E'_f)^2/2 + EI$$

$$= ZA^2 (E'_f)^2/2 + EI \quad (7)$$

The input RF power to the tank circuit can also be expressed in the form,

$$P_{RF} = (E'_f)^2/2R_z = ZA^2 (E'_f)^2/2 + EI$$

where R_z is the equivalent circuit resistance as seen by the oscillator.

$$\text{Then } 1/R_z = ZA^2 + 2\ EI/\ (E'_f)^2 \quad (8)$$

where ZA^2 is the conductance due to generating voltage and $2\ EI/(E'_f)^2$ is the conductance due to load current.

The oscillator simplified circuit is shown in Fig. 4. The oscillator equivalent circuit shown in Fig. 5 is derived from the input dc power and the output RF power formulation. The dc power input is equal to the oscillator tube losses and the delivered RF power, i.e.,

$$E_p I_p = I^2{}_p R_p + P_{RF} \quad (9)$$

$$= I_p\ (E_p - E'_f/\sqrt{2}) + E'_f)^2/2\ R_z$$

Simplifying and solving for I_p

$$I_p = E'_f/\sqrt{2}\ R_z \quad (10)$$

Substituting in Equation (9)

$$E_p = E'_f\ R_p/\sqrt{2}\ R_z + E'_f/\sqrt{2} \quad (11)$$

The oscillator conversion efficiency is defined as:

$$n = R_z/(R_z + R_p)$$

$$\text{then } R_p = R_z\ (1-n)/n$$

Substituting in Equation (11):

$$E_p = E'_f/n\sqrt{2}$$

$$= (1/A_n\sqrt{2}) \left[(K/N)\ E + I/fC_{se} \right] \quad (12)$$

The complete expression for I_p is from Equations (5) and (10)

$$I_p = (1/\sqrt{2})\ (ZA^2 E'_f + 2\ EI/E'_f)$$

$$= (A/\sqrt{2}) \{ Z\ [(K/N)\ E + I/fC_{se}]$$

$$+ 2\ EI/\ [(K/N)\ E + I/fC_{se}] \} \quad (13)$$

The input dc power to the oscillator tubes is

$$P_p = E_p I_p = (1/2n)\ \{ Z[(K/N)E + I/fC_{se}]^2 + 2\ EI \} \quad (14)$$

The total dc input power which includes the series-pass tube dissipation is:

$$P_T = I_p\ E_{dc} \quad (15)$$

Where I_p is obtained from Equation (13). In this form the oscillator tube efficiency, n, does not appear explicitly.

The circuit parameters C_T, C_{se}, C_{ac}, and K are determined from the electrode geometries of the power supply. A physical representation is shown in Fig. 6 and a schematic of the circuit is shown in Fig. 7. The surface to surface capacitances are calculated on the basis of semi-cylindrical geometry formulation and edge effects using wire to plane formulation. Since the circuit is symmetrical, both electrically and physically, an RF neutral plane exists through the center of the accelerator. Capacitances are calculated with respect to ground and the neutral plane.

The various capacitances to be considered with respect to the neutral plane are,

- C_a = Capacitance between RF electrode and corona rings surfaces.
- C_b = Total capacitance from corona rings to RF neutral plane which is the sum of C_c, C_d, and C_e.
- C_c = Total capacitance between corona rings surfaces to the beam tube surfaces.
- C_d = Total capacitance between the corona ring edges to RF neutral plane.

C_e = Twice the rectifier tube capacitance times the number of stages since the rectifier goes from corona ring to opposite corona ring.

C_f = Capacitance between RF electrode to tank surfaces.

C_g = Capacitance of RF electrode edges to vessel.

The total tank capacitance is,

$$C_T = (\tfrac{1}{2})[C_f + C_g + \frac{C_a(C_c + C_d + C_e)}{C_a + C_c + C_d + C_e}] \quad (16)$$

$$C_{se} = (2/N)\ C_a \quad (17)$$

$$C_{ac} = (C_c + C_d + C_e)/2N \quad (18)$$

$$K = 1 + 4\ C_{ac}/C_{se} = 1 + (C_c+C_d+C_e)/C_a \quad (19)$$

The formulae used to calculate the circuit capacitance from the physical geometry are [2], for semi-cylindrical geometry

$$C = 0.307 \times 10^{-12}\ S\epsilon/\log_{10}\ (r_x/r_y) \quad (20)$$

where r_x = outer radius-inches

r_y = inner radius-inches

S = length of geometry-inches

ϵ = correction factor, since electrode & corona ring shapes are not exactly semi-cylindrical.

For edge effects, a parallel-wire-to-plane formula is used,

$$C = 0.612S/\log_{10}\left[(2h/D)(1+\sqrt{1-1/(2h/D)^2}\right] \quad (21)$$

where S = length of edge-inches

h = distance between center of edge to the plane-inches

D = diameter of edge-inches

PRACTICAL APPLICATION

The power and electrical characteristics of the oscillator requirements for a new 1.5 MV, 20 mA electron accelerator were calculated from this analysis. The fixed design parameters were,

1) 1.8 MV maximum terminal voltage which fixed the corona ring to RF electrode spacing for voltage insulation.
2) 36 rectifier stages determined by the rating per rectifier.
3) 20 mA, electron beam or load current.
4) RF transformer inductance and resistance.
5) Resonant frequency of 110 to 120 kHz.
6) Diameter of rectifier column.
7) Diameter of pressure vessel which is determined from electrical spacing required for the RF electrodes, distance required for voltage insulation and rectifier column diameter.

The measured and calculated data (no-load) are shown in Figs. 8 and 9. The discrepancies in the data at low voltage are due to the RF method of lighting the rectifier tube filaments. RF current which passes through the inter-electrode capacitances of the tube is transformed to a level which can light the filament. At very low levels of RF power there is inadequate emission and therefore a loss of terminal voltage through excessive tube drop. This explains why the E_p, I_p measured characteristics do not go through zero, and why the oscillator anode power characteristic does not follow a squared function at the lower voltages.

At the present time the accelerator is being prepared for beam tests. The results of measurements under load are not available for inclusion in this paper. However, the theoretical power relationships from equations 7, 14 and 15 are shown in Fig. 10.

CONCLUSIONS

The equations give good results when used for calculating a new power supply design. The calculation of the circuit capacitances from the physical geometries is only a reasonable approximation, since the geometries, although simple in structure, represent complex capacitive elements. The lighting of the rectifier tube filaments produce additional error since they are RF heated and require a minimum level of RF power before conduction can occur. Replacement of the vacuum rectifiers with solid-state rectifiers would bring the measured performance closer to the theoretical analysis.

TABLE I. SYMBOLS AND DEFINITIONS

f	Resonant frequency - Hz
T	Period of resonant frequency
ω	Resonant frequency-radians/second
L	Inductance of RF transformer-henry
C_T	Total tank capacitance-farad
C_{se}	Coupling capacitance per rectifier stage-farad
C_{ac}	Shunt capacitance per rectifier stage-farad
K	Coupling coefficient
N	Number of rectifier stages
R_e	RF Transformer ac resistance-ohms
A	RF transformer voltage step-up ratio
n	Oscillator tube conversion efficiency
E	Terminal voltage-volts
I	Load current-amperes
E_f	RF tank voltage-volts peak between electrodes
E_f	Oscillator tube RF voltage-volts peak
E_p	Oscillator tube anode voltage-volts dc

TABLE I (Continued)

E_{dc}	Total dc voltage
I_p	Oscillator tube anode current-amperes dc
P_p	Input oscillator tube anode power-watts
P_{RF}	Input RF power to accelerator-watts
P_T	Total dc power required-watts

REFERENCES

1. M.R. Cleland and P. Farrell, IEEE Transactions on Nuclear Science NS-12, No. 3, 227 (1965).
2. Frederick E. Terman, Radio Engineers' Handbook (McGraw-Hill Book Company, N.Y., 1943).

TABLE II. DESIGN EQUATIONS

$$E_f = (K/N)E + I/fC_{se}$$

$$K = 1 + 4\, C_{ac}/C_{se}$$

$$E_f' = E_f/A$$

$$P_{RF} = ZE_f^2/2 + EI$$

$$Z = \omega^2 C_T{}^2 R_e$$

$$W^2 = 1/LC_T$$

$$E_p = (1/An\sqrt{2}) \left[(K/N)E + I/fC_{se}\right]$$

$$I_p = (A/\sqrt{2}) \left[Z \left\{(K/N)E + I/fC_{se}\right\} + 2\, EI/ \left\{(K/N)\, E + I/fC_{se}\right\}\right]$$

$$P_p = (1/n) \left[(Z/2) \left\{K/N)E + I/fC_{se}\right\}^2 + EI\right]$$

$$P_T = I_p E_{dc}$$

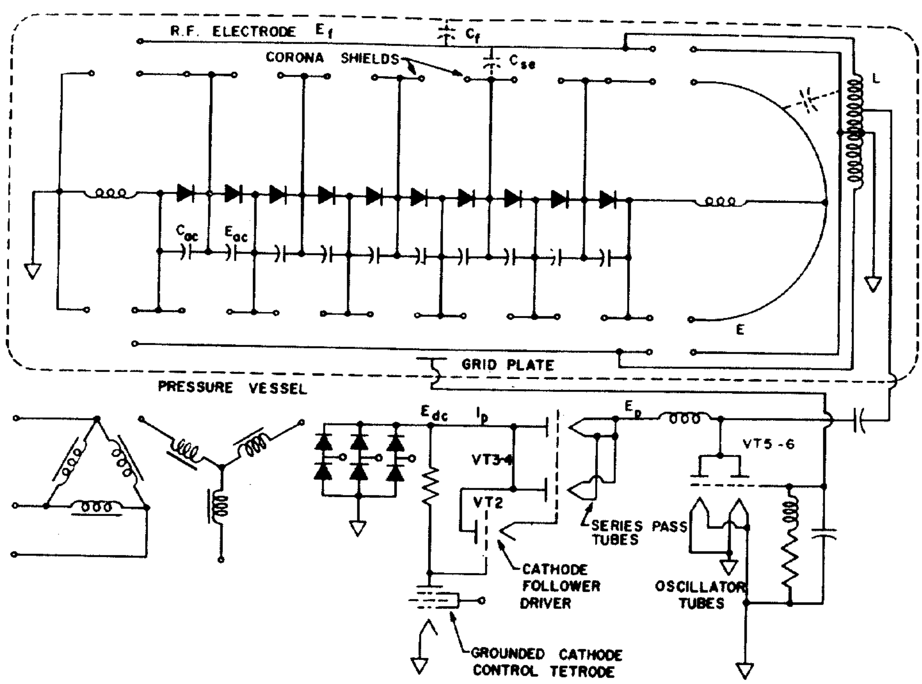

Fig. 1. High voltage generator and oscillator schematic.

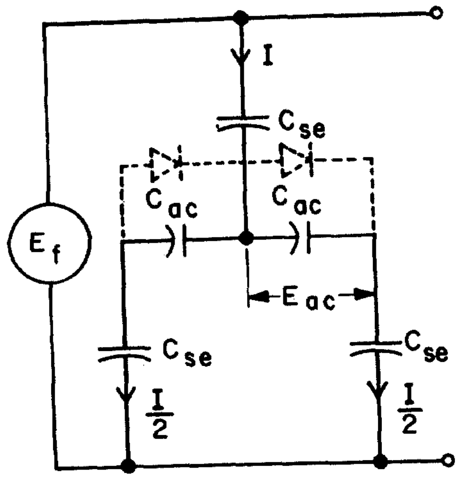

Fig. 2. Single-stage capacitive circuit.

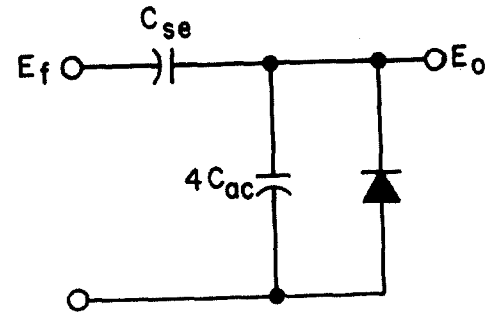

Fig. 3. Single-stage equivalent circuit.

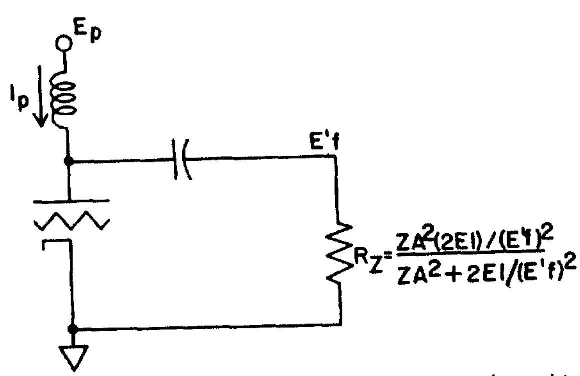

Fig. 4. Simplified oscillator circuit.

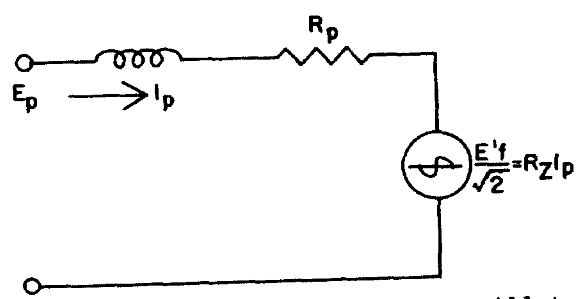

Fig. 5. Equivalent circuit of oscillator.

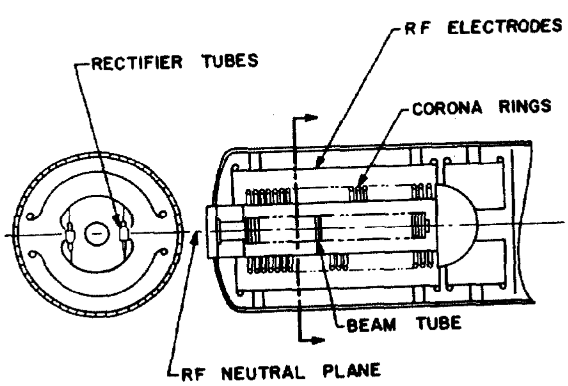

Fig. 6. Pictorial drawing of accelerator.

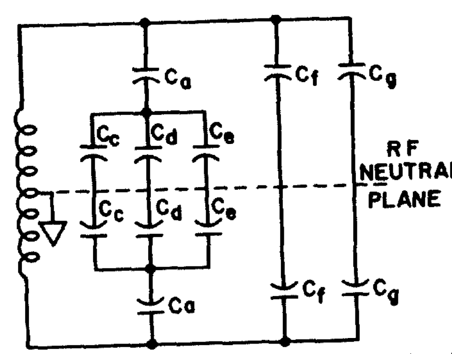

Fig. 7. Oscillator tank equivalent circuit.

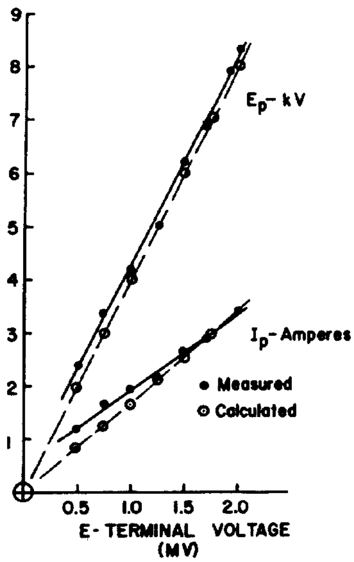

Fig. 8. Oscillator E_p-I_p characteristics-no load.

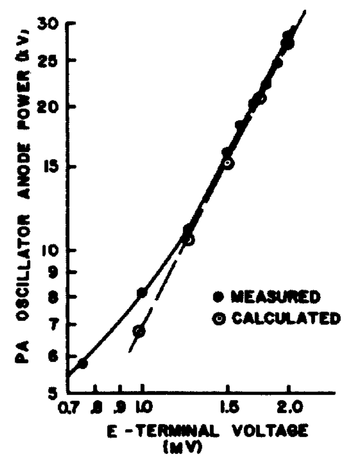

Fig. 9. Oscillator input power-no load.

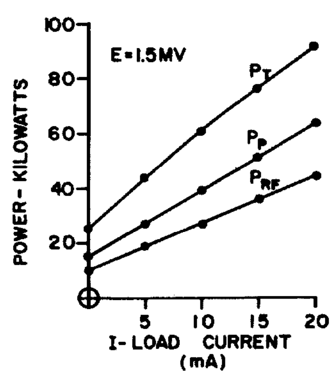

Fig. 10. Calculated load-power curves.